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Comment on "Infrared and pinching singularities in out of equilibrium QCD plasmas"

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Abstract. Analyzing the dilepton production from out of equilibrium quark-gluon plasma, Le Bellac and Mabilat have recently pointed out that, in the reaction rate, the cancellation of mass (collinear) singularities takes place only in physical gauges, and not in covariant gauges. They then have estimated the contribution involving pinching singularities. After giving a general argument for the gauge independence of the production rate, we explicitly confirm the gauge independence of the mass-singular part. The contribution involving pinching singularities develops mass singularities, which is also gauge dependent. This "additional" contribution to the singular part is responsible for the gauge independence of the "total" singular part.

In the past years, much effort has been made to incorporate quantum field theory with nonequilibrium statistical mechanics, among which, we quote those of Altherr and Seibert [1], Altherr [2], Baier et al. [3], Le Bellac and Mabilat [4] and the present author [5]. Out of equilibrium, pinching singularities appear [1] in association with self-energy inserted propagator. It has been shown in [2] that resummation of self-energy part eliminates the pinching singularity (see also [6]). An application of this result to the rate of hard-photon production from nonequilibrium quark-gluon plasmas is made in [3]. A renormalization scheme of number density is introduced in [5], such that the pinching singularities disappear. Le Bellac and Mabilat [4] are the first who have *explicitly* analyzed the infrared and mass (collinear) singularities in "lepton-pair" production rate.

First of all, let us summarize the results of [4]. The production rate of a lepton pair from a quark-gluon plasma is proportional to $\Pi(Q) \equiv -i\Pi_{12}(Q)$, where $\Pi_{12}(Q)$ is the $(1,2)$-component of the photon self-energy part in real-time massless QCD. Following [4], we deal with $\Pi(Q)$ of a scalar "photon." To two-loop order, $\Pi(Q)$ receives two contributions. The one Π_Σ comes from the diagram with self-energy inserted quark propagator and the one Π_V comes from the diagram with photon-quark vertex correction,

$$\Pi(Q) = \Pi_V(Q) + \Pi_\Sigma(Q) \tag{1}$$

$$\Pi_\Sigma(Q) = 2ie^2 \int \frac{d^4P}{(2\pi)^4} \sum_{j,\,l=1}^{2} \mathrm{Tr}\,[S_{1j}(P)\Sigma_{jl}(P) \cdot S_{l2}(P)S_{21}(P-Q)] \tag{2}$$

$$\Pi_V(Q) = -\frac{4}{3}e^2g^2 \int \frac{d^4P}{(2\pi)^4} \int \frac{d^4K}{(2\pi)^4} g^{(\mathrm{gauge})}_{\mu\nu}(K) \cdot \sum_{j,\,l=1}^{2} (-)^{j+l} \mathrm{Tr}\,[S_{1j}(P-K)\gamma^\mu S_{j2}(P) \cdot S_{2l}(P-Q)\gamma^\nu S_{l1}(P-Q-K)]\,\Delta_{lj}(K) \tag{3}$$

where

$$\Sigma_{jl}(P) = i\frac{4}{3}g^2(-)^{j+l} \int \frac{d^4K}{(2\pi)^4} g^{(\mathrm{gauge})}_{\mu\nu}(K) \cdot \gamma^\mu S_{jl}(P-K)\gamma^\nu \Delta_{jl}(K) \tag{4}$$

The (part of the) gluon propagator $\Delta_{lj}(K)$ takes the form [4]

$$\begin{aligned} \Delta_{11}(K) &= \Delta^*_{22}(K) = (1+f(K))\Delta_R(K) + f(K)\Delta_A(K) \\ \Delta_{12}(K) &= f(K)(\Delta_R(K) + \Delta_A(K)) \\ \Delta_{21}(K) &= (1+f(K))(\Delta_R(K) + \Delta_A(K)) \end{aligned} \tag{5}$$

where

$$\Delta_{R(A)}(K) = \pm i/(K^2 \pm i\eta\epsilon(k_0)) \qquad (\eta = 0^+) \tag{6}$$

The quark propagator $S_{lj}(K)$ takes the form $S_{lj}(K) = \not{K}\,\tilde{\Delta}_{lj}(K)$, where $\tilde{\Delta}_{lj}(K)$ is given by (5) with the substitution $f \to -\tilde{f}$. f ($\tilde{f}$) is related to the distribution function of gluon n (quark $\tilde{n}$) through

$$\begin{aligned} f(K) &= -\theta(-k_0) + \epsilon(k_0)\,n(|k_0|, \epsilon(k_0)\hat{\mathbf{k}}) \\ \tilde{f}(K) &= \theta(-k_0) + \epsilon(k_0)\tilde{n}(|k_0|, \epsilon(k_0)\hat{\mathbf{k}}) \end{aligned} \tag{7}$$

where $\hat{\mathbf{k}} = \mathbf{k}/k$ with $k = |\mathbf{k}|$. Form of $g^{(\mathrm{gauge})}_{\mu\nu}(K)$ in (3) and (4) depends on the gauge choice:

$$g^{(\mathrm{cov})}_{\mu\nu}(K) = g^{\mu\nu} - \alpha\frac{K_\mu K_\nu}{K^2} \qquad \text{(covariant gauge)} \tag{8}$$

$$g^{(t)}_{\mu\nu}(K) = g^{\mu\nu} - \frac{k_0}{k^2}(K_\mu n_\nu + n_\mu K_\nu) + \frac{K_\mu K_\nu}{k^2} \quad \text{(Coulomb gauge)} \tag{9}$$

where $n^\mu = (1, \mathbf{0})$. Observe that [4], in (2),

$$\sum_{j,\,l=1}^{2} S_{1j}(P)\Sigma_{jl}(P)S_{l2}(P) = F^{(n)}(P) + F^{(p)}(P) \tag{10}$$

$$\begin{aligned} F^{(n)}(P) = &-\tilde{f}(P)(\Delta^2_R(P) - \Delta^2_A(P))\not{P}\, \mathrm{Re}\Sigma_{11}(P)\not{P} \\ &- \frac{1}{2}\tilde{f}(P)(\Delta^2_R(P) + \Delta^2_A(P))\not{P} \\ &\cdot (\Sigma_{12}(P) - \Sigma_{21}(P))\not{P} \end{aligned} \tag{11}$$

$$\begin{aligned} F^{(p)}(P) = &\Delta_{\mathrm{R}}(P)\Delta_{\mathrm{A}}(P)\not{P} \\ &\cdot [(1-\tilde{f}(P))\Sigma_{12}(P) + \tilde{f}(P)\Sigma_{21}(P)]\not{P} \end{aligned} \tag{12}$$

$F^{(n)}$ is the "normal term," which is the counterpart of the one that is present in equilibrium thermal field theory (ETFT), while $F^{(p)}$ is the "pinch term," which is absent in ETFT. The last fact is due to the detailed-balance formula, which states that the ETFT counterpart of the quantity in the square brackets in (12) vanishes. In fact, as seen from (6), $\Delta_{\mathrm{R}}(P)$ $[\Delta_{\mathrm{A}}(P)]$ has poles at $p_0 = \pm p - i\eta$ $[p_0 = \pm p + i\eta]$ in a complex p_0-plane, so that the integration contour $-\infty < p_0 < +\infty$ in (2) with (12) is pinched by the poles of $\Delta_{\mathrm{R}}(P)\Delta_{\mathrm{A}}(P)$. Substituting (10) - (12) into (2), we have

$$\Pi_\Sigma(Q) = \Pi^{(n)}_\Sigma(Q) + \Pi^{(p)}_\Sigma(Q)$$

$$\Pi^{(n)/(p)}_\Sigma(Q) = 2ie^2 \int \frac{d^4P}{(2\pi)^4} \mathrm{Tr}\left[F^{(n)/(p)}(P)S_{21}(P-Q)\right] \tag{13}$$

Le Bellac and Mabilat [4] have shown that, in Coulomb gauge, the mass singularities cancel out both in $\Pi^{(n)}_\Sigma(Q)$ and $\Pi_V(Q)$, (3). While, in the covariant gauge, the cancellation holds only for $\Pi^{(n)}_\Sigma(Q)$, and in $\Pi_V(Q)$ there survives mass singularity. Then, the authors of [4] have concluded that whether or not the singularity cancellation takes place is gauge dependent. [In Appendix A, we show in a gauge-independent manner how the cancellations of mass singularities take place in $\Pi^{(n)}_\Sigma(Q)$, reconfirming the result of [4].]

We first verify that $\Pi(Q)$, (1), is gauge independent. Proof goes just as in vacuum ($T = 0$) theory. Consider the difference

$$\delta\Pi(Q) \equiv \Pi(Q)\Big|_{\text{covariant}} - \Pi(Q)\Big|_{\text{Cou}} \tag{14}$$

where "Cou" stands for Coulomb. Observe that, from (8) and (9), the difference $g^{(\mathrm{cov})}_{\mu\nu}(K) - g^{(t)}_{\mu\nu}(K)$ is proportional to K_μ and/or K_ν. Then, in evaluating $\delta\Pi(Q)$, we can use Ward-Takahashi relation,

$$\begin{aligned} S_{jk}(P-K)\not{K} S_{kl}(P) = &-i(-)^k \delta_{kl} S_{jk}(P-K) \\ &+ i(-)^j \delta_{jk} S_{kl}(P) \end{aligned}$$

with no summation over repeated indices. After doing this, we see that, among many terms in the resultant expression for $\delta\Pi(Q)$, complete cancellations occur, so that $\delta\Pi(Q)$ vanishes and then is, of course, free from mass singularities.

On the light of the above observation, let us make a closer inspection of the results of [4]. As a covariant gauge, as in [4], we take the Feynman gauge ($\alpha = 0$ in (8)) throughout in the sequel. We analyze $\Pi^{(p)}_\Sigma(Q)$ in (13) with (12). Observe first that $F^{(p)}(P) \in \Pi^{(p)}_\Sigma(Q)$ involves $\Sigma_{12(21)}(P)$. Then, (4) tells us that $\Pi^{(p)}_\Sigma(Q)$ contain $S_{12(21)}(P-K)$ and $\Delta_{12(21)}(K)$, which are proportional to $\delta((P-K)^2)$ and $\delta(K^2)$, respectively. Thus, we have $(P-K)^2 = K^2 = 0$. From $\Pi^{(p)}_\Sigma(Q)$ let us pick out a piece $\mathcal{G}^{\mu\nu} \equiv \not{P}\gamma^\mu(\not{P} - \not{K})\gamma^\nu\not{P}$ (cf. (13) with (12) and (4)). Simple algebra yields

$$g_{\mu\nu}\,\mathcal{G}^{\mu\nu} = -2P^2\not{K} \tag{15}$$

$$\begin{aligned} \delta g_{\mu\nu}\,\mathcal{G}^{\mu\nu} = &\,2P^2\frac{k_0}{k^2}\left[(2p_0 - k_0)\not{P} - p_0\not{K}\right] \\ &+ O((P^2)^2) \end{aligned} \tag{16}$$

where $\delta g_{\mu\nu} \equiv g_{\mu\nu} - g^{(t)}_{\mu\nu}(K)$. Thus, both $g_{\mu\nu}\mathcal{G}^{\mu\nu}$ and $\delta g_{\mu\nu}\mathcal{G}^{\mu\nu}$ are proportional to P^2. Then, within $\Pi^{(p)}_\Sigma(Q)$, $\Delta_{\mathrm{R}}(P)\Delta_{\mathrm{A}}(P)$ in (12) appears in a form

$$\begin{aligned} P^2\Delta_{\mathrm{R}}(P)\Delta_{\mathrm{A}}(P) &= P^2\frac{i}{P^2 + i\eta\epsilon(p_0)}\frac{-i}{P^2 - i\eta\epsilon(p_0)} \\ &= \frac{P^2}{(P^2)^2 + \eta^2} = \frac{\mathbf{P}}{P^2} \end{aligned} \tag{17}$$

This means that the pinching singularity in the "pinch" term $F^{(p)}$, (12), turns out to be a mass singularity. In fact, from the above observation leading to (15), we see that $\Sigma_{12(21)}(P)(\mathbf{P}/P^2) \ni \delta((P-K)^2)\delta(K^2)(\mathbf{P}/P^2)$, the well-known combination which leads to a mass-singular contribution. The $(P^2)^2$ term in (16) does not lead to mass-singular contribution. As in [4], let us restrict our concern to singular contributions and ignore the $(P^2)^2$ term.

As far as mass-singular contributions are concerned, above observation on the gauge independence of $\Pi(Q)$, i.e., $\delta\Pi(Q) = 0$, together with [4] $\Pi^{(n),\,\mathrm{sing}}_\Sigma\Big|_{\mathrm{Cou}} = \Pi^{\mathrm{sing}}_V\Big|_{\mathrm{Cou}} = \Pi^{(n),\,\mathrm{sing}}_\Sigma\Big|_{\mathrm{Fey}} = 0$, tells us that $\Pi^{\mathrm{sing}}_V\Big|_{\mathrm{Fey}} = -\delta\Pi^{(p),\,\mathrm{sing}}_\Sigma$, where "Fey" stands for Feynman. $\Pi^{\mathrm{sing}}_V\Big|_{\mathrm{Fey}}$ with $Q = (q_0, \mathbf{q} = \mathbf{0})$ is explicitly evaluated in [4]. As a check, in Appendix B, we evaluate $\delta\Pi^{(p),\,\mathrm{sing}}_\Sigma(q_0, \mathbf{0})$ and confirm[1] $\delta\Pi^{(p),\,\mathrm{sing}}_\Sigma = -\Pi^{\mathrm{sing}}_V\Big|_{\mathrm{Fey}}$.

The singular part of $\Pi(q_0 \equiv 2\kappa, \mathbf{0})$, being gauge independent, is also evaluated in Appendix B,

$$\Pi^{\,\mathrm{sing}} = \Pi^{(p),\,\mathrm{sing}}_\Sigma\Big|_{\mathrm{Cou}} = \Pi^{\mathrm{sing}}_V\Big|_{\mathrm{Fey}} + \Pi^{(p),\,\mathrm{sing}}_\Sigma\Big|_{\mathrm{Fey}}$$

[1] There is a missing term in $\Pi_V\Big|_{\mathrm{Fey}}$ in [4] (see Appendix B).

$$= -\frac{32}{3\pi}\alpha\alpha_s\kappa^2 \ln\frac{1}{\epsilon_y}$$
$$\cdot\int\frac{d\Omega_{\hat{\mathbf{k}}}}{4\pi}\tilde{n}(\kappa,\hat{\mathbf{k}})\tilde{n}(\kappa,-\hat{\mathbf{k}})\left[\int_{\epsilon_z}^1 dz\,\frac{(z-1)^2+1}{2z}\right.$$
$$+\int_0^\infty dz\frac{z^2+2}{z}n(\kappa z,\hat{\mathbf{k}})$$
$$\left.+\int_0^\infty dz\,\mathbf{P}\frac{z(z^2+1)}{z^2-1}\tilde{n}(\kappa z,\hat{\mathbf{k}})\right]$$
$$+\frac{16}{3\pi}\alpha\alpha_s\kappa^2\ln\frac{1}{\epsilon_y}$$
$$\cdot\int\frac{d\Omega_{\hat{\mathbf{k}}}}{4\pi}\tilde{n}(\kappa,-\hat{\mathbf{k}})\left[\int_{\epsilon_z}^1 dz\frac{(z-1)^2+1}{z}\right.$$
$$\cdot n(\kappa z,\hat{\mathbf{k}})\tilde{n}(\kappa(1-z),\hat{\mathbf{k}})$$
$$+\int_1^\infty dz\frac{(z-1)^2+1}{z}$$
$$\cdot n(\kappa z,\hat{\mathbf{k}})(1-\tilde{n}(\kappa(z-1),\hat{\mathbf{k}})$$
$$+\int_{\epsilon_z}^\infty dz\frac{(z+1)^2+1}{z}$$
$$\left.\cdot(1+n(\kappa z,\hat{\mathbf{k}}))\tilde{n}(\kappa(1+z),\hat{\mathbf{k}})\right] \quad (18)$$

Here $d\Omega_{\hat{\mathbf{k}}}$ is an element of the solid angle in a $\mathbf{k}$-space. The cutoff factor ϵ_y is defined by $1-|\hat{\mathbf{p}}\cdot\hat{\mathbf{k}}|\geq\epsilon_y$ (cf. (B.1) in Appendix B) and ϵ_z is the infrared cutoff $k\geq\epsilon_z\kappa$.

Let us clarify the relation between the present result and the result of [4]. We start with picking out from $F^{(p)}(P)$ in (12),

$$\overline{\Sigma}(P)\equiv(1-\tilde{f}(P))\Sigma_{12}(P)+\tilde{f}(P)\Sigma_{21}(P) \quad (19)$$

For the purpose of estimating $\Pi_\Sigma^{(p)}$, Le Bellac and Mabilat [4] have analyzed $\overline{\Sigma}(P)$ within the hard-thermal-loop resummation scheme. The net production rate of an (anti)quark is given by $\overline{\Gamma}(P)=\mathrm{Tr}[i\overline{\Sigma}(P)\not{P}]/(4p)$, with $P=(p,\mathbf{p})$ for quark and $P=(-p,-\mathbf{p})$ for antiquark. Arguing that $\overline{\Gamma}(P)$ on the mass shell $P^2=0$, being gauge independent, is relevant to $\Pi_\Sigma(Q)$, the authors of [4] have concluded that $\Pi(Q)$ is gauge dependent since $\Pi^{\,\mathrm{sing}}(Q)$ is. It is clear from the above argument that this is not the case. As has been discussed above in conjunction with (16), $\not{P}\delta\overline{\Sigma}(P)\not{P}$ (as well as $\not{P}\overline{\Sigma}(P)\not{P}\Big|_{\mathrm{Fey}}$) vanishes on the mass shell $P^2=0$. [As a matter of fact $\Sigma_{12(21)}(P)$ in (19) vanishes on the mass shell $P^2=0$, since P^2, K^2 and $(P-K)^2$ cannot vanish simultaneously.] However, as has been observed in (17), in calculating $\delta F^{(p)}\equiv F^{(p)}\Big|_{\mathrm{Fey}}-F^{(p)}\Big|_{\mathrm{Cou}}$ (cf. (12)), the factor P^2, (16), “eliminates” one Δ and the mass-singular contribution $\delta\Pi_\Sigma^{(p),\,\mathrm{sing}}$ (as well as $\Pi_\Sigma^{(p),\,\mathrm{sing}}\Big|_{\mathrm{Fey}}$) emerges. Thus we have learned that the mass-singular contribution does not come from the gauge-independent quantity,

$$\not{P}\overline{\Sigma}(P)\not{P}\Big|_{P^2=0}=-2ip\overline{\Gamma}(P)\not{P}\Big|_{P^2=0}\ (=0)$$

but comes from the gauge-dependent quantity

$$d\not{P}\overline{\Sigma}(P)\not{P}/dP^2\Big|_{P^2=0}=\not{P}\overline{\Sigma}(P)\not{P}/P^2\Big|_{P^2=0}$$

The observation made above in conjunction with (16) applies to the contribution (to $\delta F^{(p)}$) from the soft-K region (cf. (13) with (12) and (4)), in which $K^2\neq 0$. [The soft $(P-K)$-region is not important, at least, for the system, which is not far from thermal and chemical equilibrium.] Above observation on $\delta\Pi_\Sigma^{(p),\,\mathrm{sing}}$ holds as it is, except that $\Sigma_{12(21)}(P)$ does not vanish on the mass shell. It is to be noted in passing that $\Pi_\Sigma^{(p)}\Big|_{\mathrm{Fey}}$ develops pinch singularity. This is because, in the present case, $K^2\neq 0$, we have, in place of (15), $g_{\mu\nu}\mathcal{G}^{\mu\nu}=-2P^2\not{K}+2K^2\not{P}$. The second term on the right-hand side leads to pinching singularity in $\Pi_\Sigma^{(p)}$. Because of the factor K^2, which is small, the “residue” of the pinching contribution is relatively small.

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Appendix A: Absence of mass singularity in $\Pi_\Sigma^{(n)}(Q)$

In this Appendix, we show that $\Pi_\Sigma^{(n)}(Q)$ is free from mass singularities, reconfirming the result in [4]. Manipulation goes as follows. Substitute $F^{(n)}$, (11) with (4), into (13). Use the form of $g_{\mu\nu}^{(\mathrm{gauge})}$, (8) or (9), and forms for Δ_{lj} and S_{lj} (cf. (5)). The resultant expressions may be rearranged as

$$\Pi_\Sigma^{(n)}=\frac{8}{3\pi^2}\alpha\alpha_s\int\frac{d^4P}{2\pi}\int\frac{d^4K}{2\pi}\mathrm{Tr}\left[\mathcal{G}^{\mu\nu}(\not{P}-\not{Q})\right]$$
$$\cdot\hat{g}_{\mu\nu}^{(\mathrm{gauge})}\tilde{f}(P)(1-\tilde{f}(P-Q))\delta_\epsilon((P-Q)^2)$$
$$\cdot\left[\left(\frac{1}{2}+f(K)\right)\delta_\epsilon(K^2)\left(\Delta_R^2(P)\Delta_R(P-K)\right.\right.$$
$$\left.+\Delta_A^2(P)\Delta_A(P-K)\right)$$
$$+\left(\frac{1}{2}-\tilde{f}(P-K)\right)\delta_\epsilon((P-K)^2)$$
$$\left.\cdot\left(\Delta_R^2(P)\Delta_R(K)+\Delta_A^2(P)\Delta_A(K)\right)\right] \quad (\mathrm{A.1})$$

where $\mathcal{G}^{\mu\nu}$ is as in (15) and $\delta_\epsilon(k^2)\equiv\epsilon(k_0)\delta(K^2)$, etc. For the Coulomb gauge, $\hat{g}_{\mu\nu}^{(\mathrm{gauge})}=g_{\mu\nu}^{(\mathrm{t})}$, (9). For the covariant gauge, $\hat{g}_{\mu\nu}^{(\mathrm{gauge})}=g_{\mu\nu}+\eta K_\mu K_\nu(\partial/\partial K^2)$, where $\partial/\partial K^2$ applies to $\delta(K^2)$, $\Delta_R(K)$ and $\Delta_A(K)$. Equation (A.1) is manifestly free from mass singularities. Mass singularity arises from the terms $\Delta_{R(A)}^2(P)\Delta_{A(R)}(P-K)$ and $\Delta_{R(A)}^2(P)\Delta_{A(R)}(K)$. In obtaining (A.1), cancellations occur between those terms.

It is also obvious from (A.1) that $\Pi_\Sigma^{(n)}$ is free from divergence due to infrared singularities, provided that, as $k \to 0^+$, $f(K)$ and $\tilde{f}(K)$ in (7) meet $f(k, \hat{\mathbf{k}}) \propto k^{-n}$ with $n < 2$ and $\tilde{f}(k, \hat{\mathbf{k}}) \propto k^{-n'}$ with $n' < 2$. In actual computation of Π, for a propagator with soft momentum, one should use hard-thermal-loop resummed effective one.

Appendix B: Computation of the singular part of $\Pi_\Sigma^{(p)}(Q)$

Here, we compute the singular part of the "pinch" contribution $\Pi_\Sigma^{(\mathrm{p})}(q_0, \mathbf{q} = \mathbf{0})$. Substituting (12) with (4) into (13) and using (15) and (16), and the forms for Δ_{lj} and S_{lj}, we have

$$\begin{aligned}\Pi_\Sigma^{(p),\,\mathrm{sing}} = &-\frac{128}{3\pi}\alpha\alpha_s Q^2 \int \frac{d^4P}{2\pi} \int \frac{d^4K}{2\pi} G^{(\mathrm{gauge})} \\ &\cdot(1 - \tilde{f}(P-Q))\delta_\epsilon(K^2)\delta_\epsilon((P-K)^2) \\ &\cdot\delta_\epsilon((P-Q)^2)\,\mathbf{P}\frac{1}{P^2}[f(K)\tilde{f}(P-K) \\ &-\tilde{f}(P)(1 + f(K) - \tilde{f}(P-K))]\end{aligned}$$

where

$$G^{(\mathrm{Fey})} = \frac{k_0}{q_0}$$

$$G^{(\mathrm{Cou})} = \frac{(q_0 - k_0)^2 + k_0^2}{2q_0k_0}$$

Here use has been made of $p_0 = q_0/2$, which comes from $(P-Q)^2 = 0$. Making the change of variable $P \to P + K$, we extract the mass-singular part,

$$\begin{aligned}&\delta(P^2)\int d^4K\delta(K^2)\,\mathbf{P}\frac{1}{(P+K)^2} \\ &= \pi\delta(P^2)\int dk\,k^2 \int dk_0\,\delta(K^2) \\ &\quad\cdot\int_{-1+\epsilon_y}^{1-\epsilon_y} d(\hat{\mathbf{p}}\cdot\hat{\mathbf{k}})\frac{1}{p_0k_0 - \mathbf{p}\cdot\mathbf{k}} \\ &\to \frac{\pi}{p}\delta(P^2)\int dk\,k \int dk_0\,\epsilon(p_0k_0)\delta(K^2)\ln\frac{1}{\epsilon_y} \quad \text{(B.1)}\end{aligned}$$

Using (7), cutting off the infrared region, $k\ (\equiv \kappa z) \geq \epsilon_z\kappa$, and changing the integration variable suitably, we arrive at the final form. As it should be, $\delta\Pi_\Sigma^{(p),\,\mathrm{sing}} = \Pi_\Sigma^{(p),\,\mathrm{sing}}\Big|_{\mathrm{Fey}} - \Pi_\Sigma^{(p),\,\mathrm{sing}}\Big|_{\mathrm{Cou}}$ is equal to $-\Pi_V^{\mathrm{sing}}$, which has been evaluated in [4]. There is a missing term though in Π_V^{sing} in [4],

$$-\frac{32}{3\pi}\alpha\alpha_s\kappa^2(\tilde{n}(\kappa))^2\ln\frac{1}{\epsilon_y}\int_{\epsilon_y}^1 dz\,\frac{1-z}{z}$$

where $\kappa = q_0/2$. The form for $\Pi^{\,\mathrm{sing}}$, being gauge independent, reads (18) in the text.

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